1. Compute the smallest positive integer n such that $\dfrac{n}{2}$ is a perfect square and $\dfrac{n}{3}$ is a perfect cube.

   Answer: 648

   Solution 1: If n is minimal, then only 2 and 3 can divide n, so $n = 2^a 3^b$. Therefore, $a \equiv 1 \pmod 2$ and $a \equiv 0 \pmod 3$, so $a = 3$. Furthermore, $b \equiv 0 \pmod 2$ and $b \equiv 1 \pmod 3$, so $b = 4$. Therefore, $n = 2^3 \cdot 3^4 = \boxed{648}$.

   Solution 2: We know the integer $n = 2r^2 = 3s^3$ for some integers r, s. Then we have $3 \mid r^2$, so $3 \mid r$ and $9 \mid 2r^2$. It follows that $9 \mid 3s^3$, so $3 \mid s$. Similarly, we have $2 \mid s$ and $6 \mid s$. We observe that when $s = 6$, $648 = 2 \cdot 18^2 = 3 \cdot 6^3$, and our answer is $\boxed{648}$.

2. On a certain planet, the alien inhabitants are born without any arms, legs, or noses. Every year, on their birthday, each alien randomly grows either an arm, a leg, or a nose, with equal probability for each. After its sixth birthday, the probability that an alien will have at least 2 arms, at least 2 legs, and at least 1 nose on the day is $\frac{m}{n}$, where m and n are relatively prime positive integers. Compute $m + n$.

   Answer: 313

   Solution: Each ordered 6-tuple of A's (arms), L's (legs), and N's (noses) represents a possible growth history of a 6 year old alien. There are $3^6 = 729$ possible 6-tuples. To satisfy the given conditions, note that an alien must have either 3 arms, 2 legs, and 1 nose; 2 arms, 3 legs, and 1 nose, or 2 arms, 2 legs, and 2 noses, so we wish to find the total number of permutations of $AAALLN$, $AALLLN$, and $AALLNN$, and divide it by the total number of 6-tuples. There are $\frac{6!}{3!2!1!} = 60$ permutations of both $AAALLN$ and $AALLLN$, and $\frac{6!}{2!2!2!} = 90$ permutations of $AALLNN$. So the probability is $\frac{60+60+90}{729} = \frac{70}{243}$ and our answer is $\boxed{313}$.

3. Three distinct integers a_1, a_2, a_3 between 1 and 21, inclusive, are selected uniformly at random. The probability that the greatest common factor of $a_i - a_j$ and 21 is 7 for some positive integers i and j, where $1 \leq i \neq j \leq 3$, can be written in the form $\frac{m}{n}$, where m and n are relatively prime positive integers. Compute $m + n$.

   Answer: 49

   Solution: Since all three of the integers are distinct, the greatest common factor of $a_i - a_j$ and 21 is 7 when a_i and a_j are congruent modulo 7. The probability that all three integers are distinct modulo 7 is $\frac{18}{20} \cdot \frac{15}{19} = \frac{27}{38}$, so the complement of this is the probability that we want, or $\frac{11}{38}$. Therefore, our answer is $\boxed{49}$.